which shows that in the example in question instability can occur in the fields $H > p\gamma_2\rho a_5{}^2/K_0T \sim 10^5$ gauss.

The author expresses his gratitude to L. I. Sedov for the attention shown to this work and for valuable comments made during its assessment.

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PLANE ELECTROHYDRODYNAMIC FLOW WITH REVERSE CURRENT

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We consider the modes of flow of unipolarly charged jets in the case when the charged particles return along the peripheral zones of the hydrodynamic streams to the electrode-"emitter" under the action of both the induced and the external electic field. It is shown that the reverse current increases with increasing width of the section through which the charged particles enter, the electric charge density at this section and the intensity of the external retarding field. The electric current from the electrode emitter is reduced by the reverse current and the retarding effect of the external field.

During the present investigation we solve the two-dimensional electrohydrodynamic equations numerically, determine the local electrical parameters and the character of the flow established over the whole region.

The reserve currents result from the spatial (two-dimensional) character of the real electrohydrodynamic flows. The effect becomes of interest when constructing electrohydrodynamic energy transducers where the losses caused by the fact that a part of the current returns to the emitter along the boundary layer must be taken into account, and in technological processes such as electrostatic spraying and dusting. The retarding influence of the external electric field on the charged particles becomes of considerable importance in the cleaning processes where the reverse currents are of additional advantage.

Certain papers dealing with plane flows are given in [1 – 4].

1. We consider a steady plane flow between two infinite parallel electrodes $x^\circ = 0$ (emitter) and $x^\circ = L$ (collector), permeable to the fluid (the superscript ° denotes dimensional quantities). An external electric field which is generated between these electrodes hinders the motion of the electrically charged particles in the direction in which they are transported by the flow of fluid. A stream of a viscous incompressible fluid propagating through the interelectrode space in the direction of the x°-axis expands in the downstream direction from its initial cross section H (at $x^\circ = 0$). The longitudinal of velocity components u° varies across the stream from its maximum value at the axis to zero at the stream boundary. The electrical-hydraulic intercation parameter is assumed small, therefore the velocity field is specified in its first approximation.

The electrical charges generated in the region $x^\circ < 0$ (using e.g. a corona discharge) are carried by the fluid into the region $0 < x^\circ < L$. The size of the cross section through which the charged particles enter, satisfies the inequality $h < H$, and the charge density q_* is assumed constant and known at this cross section. Let us assume for definiteness that the particles have a positive charge, i.e the charge density $q^\circ > 0$, and the electrode $x^\circ = 0$ is earthed. Then the positive electric potential φ°_+ of the electrode $x^\circ = L$ corresponds to the external electric field retarding the particles.

The interaction between the charges and the fluid leads to formation of an "electric" jet (a zone in which $q^\circ \neq 0$) [1] in the interelectrode space. The induced transverse field deflects the particles towards the peripheral zones of the fluid flow where the longitudinal rate of drift of the charges is smaller. As long as the axial component of the current density $j_x^\circ = q^\circ (u^\circ - b\partial\varphi^\circ / \partial x^\circ) > 0$ (b is mobility), the charges move downstream along with the flow. If $j^\circ_x < 0$, a reverse current appears, i.e. the particles return to the electrode emitter. It is clear that inequality $\partial\varphi^\circ / \partial x^\circ > 0$ represents the necessary condition for the reverse current to exist, i.e. a reverse current may be generated only in the region in which the potential increases in the longitudinal direction.

The author of [1] dealt with cases when no reverse currents appeared, although the electric jet width could, in certain zones, exceed that of the hydrodynamic stream. Below we investigate the condition under which flows with reverse currents take place. The presence of a reverse current implies reduces collector current. Another factor affecting the magnitude of this current is the influence of the negative external field which tends to reduce the speed of the charged particles evertwhere. The relations connecting these two factors which together determine the reduction in the emitter current, are discussed below.

2. Let us pass to dimensionless variables in accordance with the following formulas (using the accepted notation):

$$\varphi^\circ = \frac{v_* H}{b}\varphi, \qquad q^\circ = \frac{\varepsilon v_*}{4\pi b H} q, \qquad \mathbf{j}^\circ = \frac{\varepsilon v_*^2}{4\pi b H}\mathbf{j} \tag{2.1}$$

$$\mathbf{V}^\circ = v_* \mathbf{V}, \qquad x^\circ = Hx, \qquad y^\circ = Hy$$

Under the assumptions made and in the variable given above, the equations describing the processes under investigation become [1]

$$\Delta\varphi = -q \tag{2.2}$$

$$\frac{\partial q}{\partial x}\left(u - \frac{\partial \varphi}{\partial x}\right) - \frac{\partial q}{\partial y}\cdot\frac{\partial \varphi}{\partial y} = -q^2, \qquad \mathbf{j} = q(\mathbf{V} - \operatorname{grad}\varphi)$$

$$\left(u = \sqrt{\frac{3}{3+x}}\left[1 - \left(\frac{2y}{1+0.3x}\right)^{3/2}\right]^2\right)$$

The choice of the distribution of the longitudinal velocity component u (given in brackets in (2.2)) allows for the fact that the velocity becomes zero at the boundary of the hydrodynamic stream $Y = (1 + 0.3\ x)\ /\ 2$. The transverse velocity component is assumed to be zero everywhere. Since the flow is symmetric about the axis $y = 0$, it is sufficient to consider the region $y \geqslant 0$ only, taking into account the symmetry condition that $\partial\varphi\ /\ \partial y = 0$ for $y = 0$. It is also clear that $\varphi \to \varphi_+ x$ as $y \to \infty$. Thus the boundary conditions for (2.2) have the form

$$\varphi = 0 \ \text{ for } \ x = 0, \ \varphi = \varphi_+ \ \text{ for } \ x = l$$

$$\partial\varphi\ /\ \partial y = 0 \ \text{ for } \ y = 0, \ \varphi \to \varphi_+ x \ \text{ as } \ y \to \infty$$

$$q = \beta \ \text{ for } \ x = 0, \ y \leqslant a\ /\ 2$$

$$\left(a = \frac{h}{H}, \ l = \frac{L}{H}, \ \beta = q_* \frac{4\pi b H}{\varepsilon r_*}, \ \varphi_+ = \varphi_+^\circ \frac{b}{v_* H}\right)$$

We note that the assumption that a surface charge is absent from the boundary Γ of the electric jet at which the charge density undergoes a discontinuity.

The system (2.2) must be solved separately for the regions where $q = 0$ and for the regions where $q \neq 0$, matching the solutions at the boundary Γ. The requirement that the potential and its first order derivatives are all continuous serves as the condition for matching the solutions. The problem has four characteristic parameters: the geometrical factors a and l, and the quantities β and φ_+ ; but an arbitrary combunation of these factors does not correspond to a flow with reverse current.

The characteristic features of the flow which are important in practice and in which the effects under consideration will appear, are the total currents: the total current J_+ passing through the collector and the total current J_0 emerging into the interelectrode space

$$J_0 = \int_{-a/2}^{a/2} j_x(0, y)\, dy, \qquad J_+ = \int_{-\infty}^{\infty} j_x(1, y)\, dy \tag{2.3}$$

A numerical method was used to solve the problem, namely the method of consecutive approximations [1] based on consecutive integration of the elliptic and the hyperbolic equation of (2.2). The elliptic equation was integrated using the Seidel method of consecutive displacements and the hyperbolic equation by the method of characteristics for which the following relation holds:

$$\frac{dx}{u - \partial\varphi\ /\ \partial x} = \frac{dy}{-\partial\varphi\ /\ \partial y} = \frac{dq}{-q^2} \tag{2.4}$$

In the course of solution we required that the boundary condition holds, as $y \to \infty$, at sufficiently large but finite distances from the axis $y = 0$. It was also assumed that $l = 1$, $a < 1$ and the parameter β varies from zero to a sufficiently large value, up to the emergence into the "saturation" mode.

3. Let us consider the case when the external retarding force is absent, i.e. $\varphi_+ = 0$. The decrease in the value of J_+ (relative to J_0) is caused by the reverse current generated by the induced electric fields only.

The deformation of the region in which $q \neq 0$ exhibits the following distinctive features when $\beta \rightarrow \infty$. When $\beta = 0$, the electric jet maintains its width along its whole length. When β increases, the rate at which the jet widens also increases and the interelectrode space within the flow begins to be filled with charges more and more completely. Finally a zone of reverse current appears, approaching the electrode $x = 0$ at large y. With further increase in the values of β the zone widens, extending in the direction of the axis of the flow. At sufficiently large β the boundary Γ ceases to vary and the saturation mode is reached.

Figure 1 depicts a family of curves Γ for $a = 0.4$ and various values of β. A zone S, in which $q \equiv 0$, lies to the left of each curve, and a zone in which $q \neq 0$, to the right of each curve. The dash-dot line represents the boundary of the hydrodynamic stream Y.

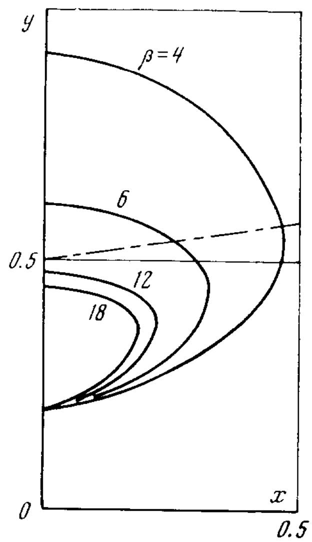

Fig. 1

When the values of the parameter a are large, the reverse current appears earlier relative to β since the charge emitted at the given value of β is greater and this produces a larger induced field. When $a = 0.6$, the reverse current first appears at β of about 0.7, while for $a = 0.4$ the corresponding value is 3.5.

In the saturation mode the zone S decreases with increasing a, and a value $a = a^*$ exists at which the region S degenerates into the point $x = 0$, $y = a^*/2$ and the charged particles fill the interelectrode space completely. The quantity a^* determines the largest usable segment of the electrode $x = 0$ in the saturation mode, i.e the zone through which particles emerge into the working space $0 < x < 1$. This corresponds to restricting the size of the cross section in the plane $x = 0$ where the density of the charged particles is specified. In our computations we have adopted $a^* \approx 0.5$.

A flow with reverse current in the absence of an external field is depicted on Fig. 2a where the characteristics (the trajectories of motion of the charged particles) for $\beta = 6$ and $a = 0.4$ are shown by dashed lines. A neutral characteristic exists, going to infinity and dividing the region of flow into two parts, one of which corresponds to the particles returning to the emitter, and the other to the particles arriving at the collector.

Figure 2b shows transverse distributions of the charge density at various cross sections $x_0 = \text{const}$ (curves *1*—*3* correspond to $x_0 = 0.1$, 0.5 and 0.9) for $\beta = 6$ and $a = 0.4$. At the cross section $x_0 = 0.1$ which intersects the zone S, the charge density undergoes a discontinuity twice. In each cross section q decreases asymptotically to zero as $y \rightarrow \infty$. We note that although the charged particles occupy almost the whole space including infinity, their peripheral density is not high.

Computation of the total quantity of the reverse current $|J_-|$ ($J_- < 0$ is the current passing through the electrode $x = 0$ in the peripheral zone) shows that it increases with increasing β and a. Figure 3 shows $J_0(\beta)$ and $J_+(\beta)$ plotted for $a = 0.4$. For the values of the parameter β at which the reverse current is zero, $J_0 = J_+$. If the reverse current is not zero, then $J_0 - J_+ = |J_-|$. The relationships given show that the currents J_0 and J_+ practically cease to grow (the saturation mode) even for $\beta \approx 20$. The reverse current also reaches a maximum and represents not more than 5% of J_0.

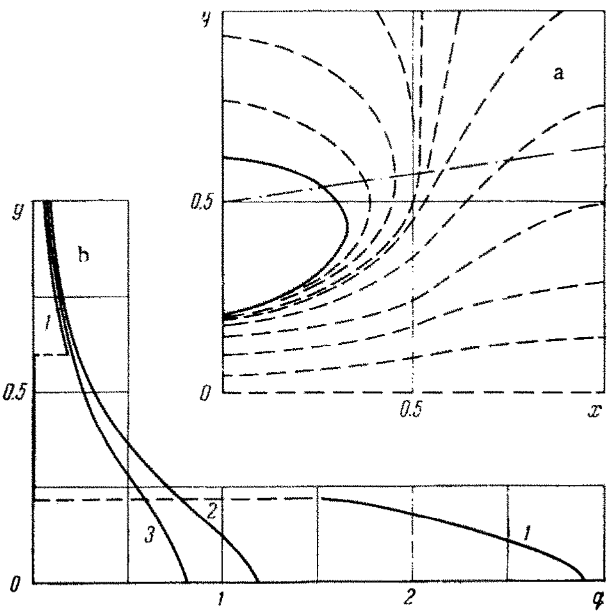

Fig. 2

It should be noted that a similar small current passes also through the collector at its peripheral sections. The current passing through the electrode $x = 1$ is basically determined by the magnitude of the current through the neutral zone (at the axis $y = 0$) the size of which corresponds approximately to the size of the cross section of the hydrodynamic stream at $x = 1$.

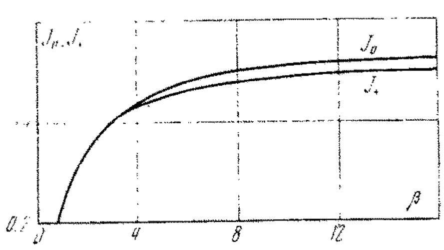

Fig. 3

It appears that the reverse current increases when $l > 1$ and the external retarding field is not zero; we consider this aspect below.

4. The presence of an external retarding field ($\varphi_+ > 0$) leads on one hand to a general reduction in the velocity of motion of the charged particles, with consequent decrease in the current J_0. This phenomenon can be called the source "supression" effect. On the other hand, the retarding field assists the earlier appearance of the reverse current. This is connected with the increase in the size of the zone in which $\partial\varphi/\partial x > 0$.

Utilizing the computations made and analysis of the system of equations performed, we arrive at the following picture of the deformation of the region occupied by the electric jet.

Let us suppose that at $\varphi_+ = 0$ the reverse currents are absent and the electric jet boundary appears within the hydrodynamic region of flow (e. g when $a = 0.4$ and $\beta = 2$). On increasing the value of φ_+ the jet spreads and its boundary Γ intersects the plane $x = 1$ at the points N situated further and further from the axis of the flow. This upward motion of the point N along the straight line $x = 1$ continues until the condition $u - \partial\varphi / \partial x = 0$ is reached (up to this instant the flow has no reverse current), Such a point $N = P\ (1, y_0)$ at which the relations

$$u - \partial\varphi / \partial x = 0, \qquad \partial\varphi / \partial y = 0 \tag{4.1}$$

both hold simultaneously, represents a singularity of the characteristic equations (2.4).

To determine the type of this singularity let us write the characteristic equation in the form of an equivalent autonomous system of second order equations (in which t is a parameter)

$$dx / dt = u - \partial\varphi / \partial x, \quad dy / dt = -\partial\varphi / \partial y \tag{4.2}$$

for which the point $P\ (1, y_0)$ is the equilibrium position. We now expand the function φ near the point in question, into the series

$$\varphi = \sum_{n,m=0}^{\infty} a_{nm} (x-1)^n (y-y_0)^m, \quad a_{0m} = 0 \quad \text{for } m \geqslant 2 \tag{4.3}$$

$$(|x-1| < 1, |y-y_0| < 1)$$

(The relation given above for a_{0m} follows from the second equation of (4.1), which holds for any y when $x = 1$).

Substituting (4.3) into (4.2) and passing to new small variables $x - 1 = \xi$ and $y - y_0 = \eta$, we linearize the system, with (4.1) taken into account. This gives us the following relations near the equilibrium position:

$$\frac{d\xi}{dt} = -2a_{20}\xi - a_{11}\eta, \qquad \frac{d\eta}{dt} = -a_{11}\xi$$

The roots of the characteristic equation defining the eigenvalues of the matrix of this system are real (a_{nm} are real) and of different signs. Consequently the equilibrium point (point P) is a saddle point [5].

From the point P a reflected characteristic proceeds to its intersection with the plane $x = 0$. This characteristic represents the outer boundary Γ_+ of the electric jet, above which $q \equiv 0$. Thus, the interelectrode space occupied by the charged particles is now bounded. We have, just as in Sect. 3, a boundary Γ to the left of which $q = 0$. A further increase in the value of φ_+ is accompanied by a descent of the boundary Γ_+ towards the axis $y = 0$ and a deformation of Γ.

Figure 4 shows the boundaries Γ_+ (dashed lines) and Γ (solid lines) of the electric jet for $a = 0.4$, $\beta = 2$ and various values of φ_+ (the dash-dot line represents the hydrodynamic stream boundary). When φ_+ increases, the region of flow decreases and the zone of reverse current increases. At some value $\varphi_+ = \varphi_+^*$, a part of the charged particles emerging from a (central) sector $0 \leqslant y \leqslant a/2$ of the emitter, begins to return to another (upper) sector. Such an upper sector must become disconnected and therefore this reduces the unstable area of the emitter. In the limit the whole emitter must become disconnected (total suppression of the emitter).

Fig. 5 depicts the pattern of flow for $a = 0.4$, $\beta = 2$ and $\varphi_+ = 0.08$, and the dashed lines represent the trajectories of motion of the charged particles. A neutral characteristic which arrives at the singularity, exists and forms together with Γ_+ and Γ the boundary for the zone of reverse current. It is obvious that turning of the particles towards the electrode $x=0$ is realized also near the collector. We note that the charge

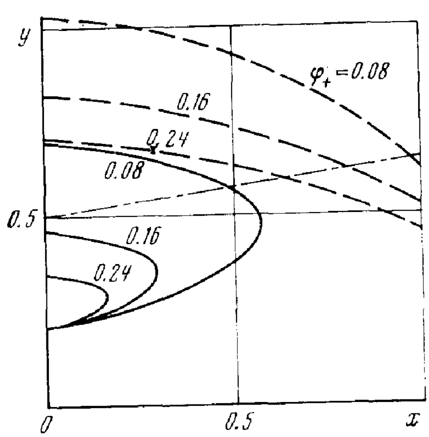

Fig. 4

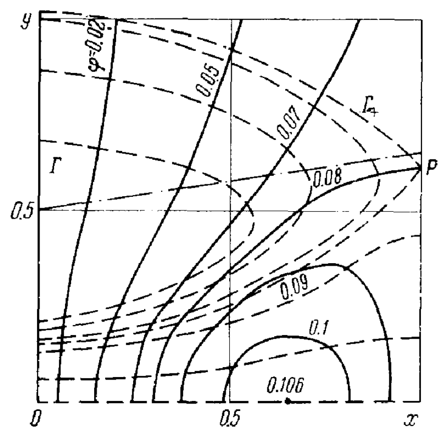

Fig. 5

density undergoes a discontinuity at the boundary Γ_+. The equipotential lines shown on Fig. 5 (solid lines) illustrate the action of the electrical forces on a particle ($a = 0.4$, $\beta = 2$, $\varphi_+ = 0.08$).

Figure 6 shows how the integral parameters J_0 and J_+ given by the formulas (2.3) vary with the retarding potential φ_+, for the case $a = 0.4$, $\beta = 2$. It is essential that $J_0(\varphi_+)$ and $J_+(\varphi_+)$ are both decreasing functions. The decrease in J_0 is caused by suppression of the emitter, and the decrease in J_+ is due to the combined effect of suppression and reverse current. For the values of φ_+ at which the reverse current is zero, $J_0 = J_+$. The reverse current $J_0 - J_+$ increases with increasing φ_+ and reaches the proportion of 23% J_0 when $\varphi_+ = 0.32$. The results obtained indicate that when $\varphi_+ \neq 0$, the reverse current assumes an extraordinary importance. All the same, the effect of emitter suppression remains the main factor causing a decrease in J_0. For example, when $\varphi_+ = 0.32$, the combined effect reaches 75%, while 54% are due to suppression.

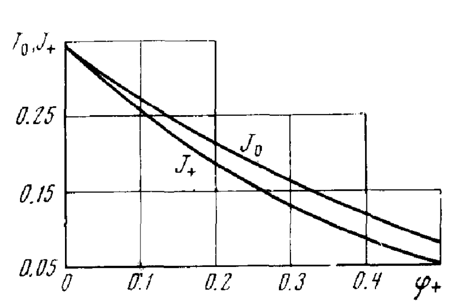

Fig. 6

In conclusion we mention the unusual course of deformation of the region of flow with increasing φ_+ in the case when a reverse current exists even when $\varphi_+ = 0$ (e.g. when $a = 0.4$, $\beta = 4$). When φ_+ increases, the region of flow varies from one infinitely large as the singularity first appears at infinity, to one localized in the central part at $y = 0$.

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WAVES OF IONIZATION AND RECOMBINATION IN A WEAKLY IONIZED MAGNETIZED PLASMA

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It is established that several types of the ionization and recombination waves may occur in a weakly ionized plasma containing additions of an easily ionizable component, in the presence of a magnetic field. The rate of propagation of such waves are determined by the conditions arising from the fact that the waves have certain structures. It is shown that the waves considered in the present paper satisfy the criterion of evolutionarity.

The problem of propagation of an ionization wave through a weakly ionized plasma in the absence of a magnetic field was first posed in [1]. The expression obtained there for the rate of propagation of the ionization wave was confirmed experimentally in [2], but at the same time the experimental data obtained in [3, 4] disagreed with the results of [1]. The analysis in [1] was performed for a model of two-fluid hydrodynamics and it was assumed that the plasma is in the state of ionization equilibrium. A motion of the ionization wave during which this equilibrium is disturbed, was studied in [5]. A study of